Leamington Federation
Sydenham Primary School and Lighthorne Heath
Mathematics Calculation Policy

Introduction:

The calculation policy has been written by the maths co-ordinator with support from school staff. On recommendations from Maths subject leader development meetings, Wandsworth LA Calculation policy has been used to support our school approach.

This policy is divided into sections for each operation (addition, subtraction, multiplication and division) and each year group. It also outlines the skills needed to access the 'Mastery' level for each year group.

The policy outlines expectations for both mental and written calculations and includes statements from the National Curriculum:

- National curriculum statutory statements are in **bold**
- National curriculum non-statutory guidance are in *italics*

The policy also outlines expectations from the Early Years Foundation Stage including exemplification materials. For more detail or information on this please see the gov.uk website.

This calculation policy is designed to ensure a consistent approach to calculation methods across the whole school. Teachers are advised to support children's understanding of a particular method before introducing them to the next stage.

Mastery

Progress in mathematics should be assessed according to the extent to which pupils are gaining a deep understanding of the content taught for that year. The essential idea behind mastery is that all children need a deep understanding of the mathematics they are learning so that future mathematical learning is built on solid foundations which do not need to be retaught.

Within this policy, mastery at each level and operation is included. Please see appendix 1 Teaching for Mastery for example questions and how to adapt the curriculum to extend pupils working at depth in mathematics.

Maths Hub

Teachers use the Maths Hub and access resources from the National Centre for Excellence in the Teaching of Mathematics (NCETM) website. This website has a wealth of resources available to support all areas of mathematics teaching, including mastery. These activities are linked to the National Curriculum.

Early Years: Addition

Development Matters 40-60

- Recognise some numerals of personal significance.
- Recognises numerals 1 to 5.
- Counts up to three or four objects by saying one number name for each item.
- Counts actions or objects which cannot be moved.
- Counts objects to 10, and beginning to count beyond 10.
- Counts out up to six objects from a larger group.
- Selects the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Counts an irregular arrangement of up to ten objects.
- Estimates how many objects they can see and checks by counting them.
- Uses the language of 'more' and 'fewer' to compare two sets of objects.
- Finds the total number of items in two groups by counting all of them.
- Says the number that is one more than a given number.
- Finds one more or one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting. Records, using marks that they can interpret and explain.
- Begins to identify own mathematical problems based on own interests and fascinations.

Early Learning Goal

Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

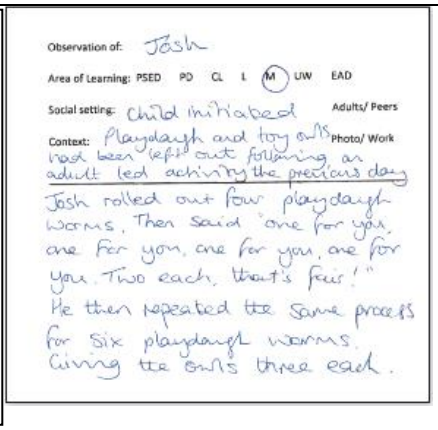
Representations to support

Notes:

Here is Hattie's leg! She counted how many stripes there were on her tights. Hattie counted 8 stripes and she told me that if she showed one more then there would be 9 stripes.

Next steps:

If you covered up one stripe then there would be one less. How many is that?



Observation of: Josh

Area of Learning: PSED PD CL L (M) UW EAD

Social setting: Child initiated Adults/ Peers

Context: Playdough and toy owls Photo/ Work
had been left out following an
adult led activity the previous day

Josh rolled out four playdough worms. Then said "one for you, one for you, one for you, one for you. Two each, that's fair!" He then repeated the same process for six playdough worms giving the owls three each.

Mastery

Children estimate a number of objects and check quantities by counting up to 20. They solve practical problems that involve combining groups of 2,5,10 or sharing into equal groups.
Use these resources to support children in achieving 'Exceeding':
https://nrich.maths.org/early-years

Year 1: Addition

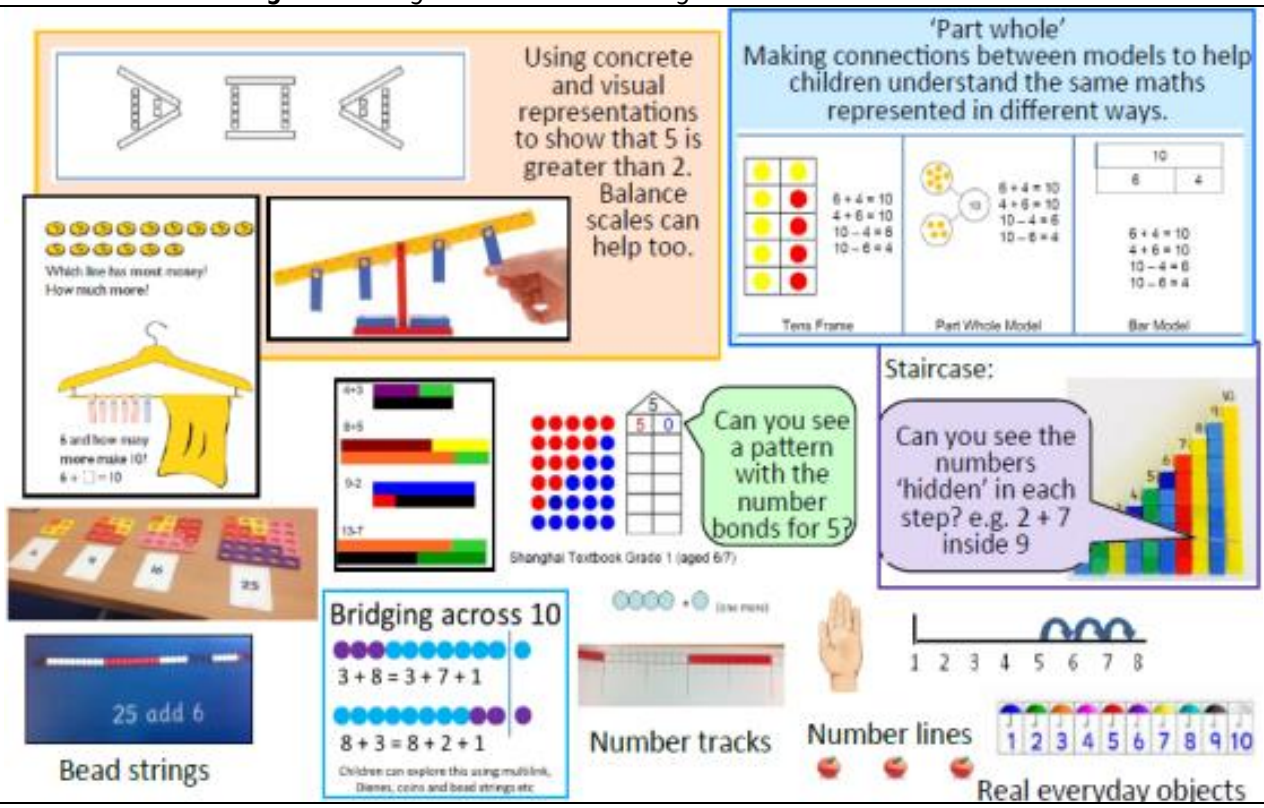
Mental calculations

- **Read, write and interpret mathematical statements using symbols + - =**
- **Represent and use number bonds and related addition facts within 20**
- **Add one digit and two digit numbers up to 20, including zero**
- **Solve one step problems using concrete objects and pictorial representations, and missing number problems such as 7= ? – 9**
- **Use the language one more and one less**

Written calculations

- Begin to compare (same/different) for commutative calculations
  e.g. 5 + 3 = 3 + 5
- Be able to partition numbers in order to help breakdown calculations
- *Memorise and reason with number bonds to 10 and 20 in several forms*
- Add using objects, numicon, cubes etc. and number lines and tracks
- Check with everyday objects
- Ensure pre-calculation steps are understood, including:
  - Counting objects (including simple concrete problems)
  - Conservation of number e.g. arranging the number of objects in different ways doesn't change to number of objects)
  - **Recognise place value in numbers beyond 20**
  - **Counting** as reciting and as enumerating

Representations to support



Mastery

Relating numbers to five and ten helps develop knowledge of the number bonds within 20. For example given 8 + 7, thinking of 7 as 2 + 5 and adding the 2 to 8 to make 10 and then the 5 to total 15.

Thinking of part- whole relationships is helpful in linking addition and subtraction.

Year 2: Addition

Mental calculations

- **Add numbers including concrete objects, pictorial representations, and mentally, including:**
  - **A two-digit number and ones**
  - **A two-digit number and tens**
  - **Two two-digit numbers**
  - **Adding three one-digit numbers**
- **Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100**
- **Demonstrate the commutative law of addition**

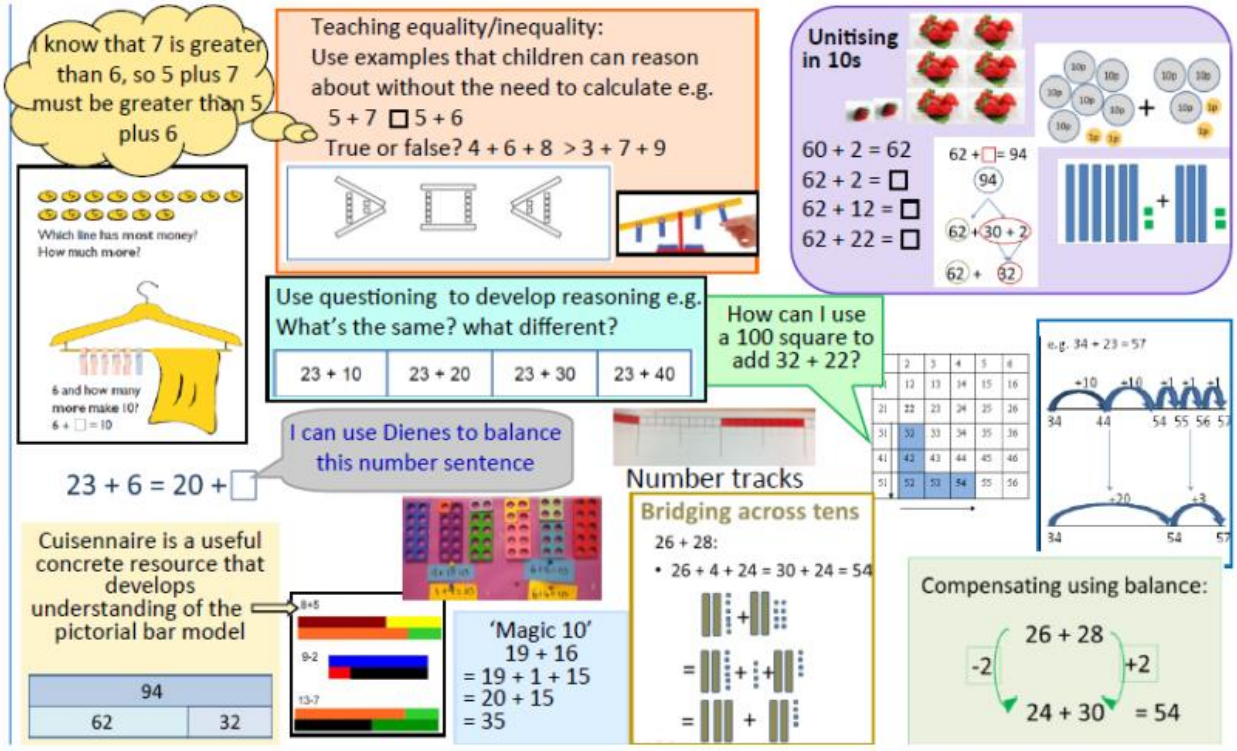
Written calculations

- Partition numbers in different ways (see picture)

65 = 60 + 5
65 = 50 + 15
65 = 40 + 25
65 = 30 + 35
65 = 20 + 45
65 = 10 + 55

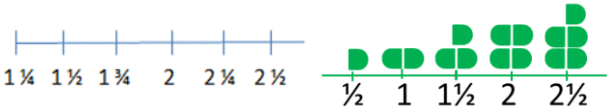
- Bridge across 10 using knowledge of partitioning
- Use a hundred square
- *Check calculations using the inverse and by adding numbers in different orders*
- *Begin to record addition in columns to support place value and prepare for formal written methods with larger numbers*

Representations to support



Fractions

Pupils should count in fractions up to 10 starting from any number and using the equivalence on the number line (for example, 1 ¼ 1 ½ 1 ¾ 2).



Mastery

Understanding that addition of two or more numbers can be done in any order.
When adding two numbers it can be more efficient to put the larger number first e.g., given 3 + 8 it is easier to calculate 8 + 3.
When adding three or more numbers it is helpful to look for pairs of numbers that are easy to add.
Understand the importance of the equal sign meaning 'equivalent' to is crucial for later work in algebra. Empty box problems can support the development of this key idea. Altering where the equals sign is placed develops fluency and flexibility.

Year 3: Addition

Mental calculations

- **Add numbers mentally including:**
  - **A three-digit number and ones**
  - **A three-digit number and tens**
  - **A three-digit number and hundreds**
- Partition all numbers and recombine. Start with TU+TU the HTU+TU
- Bridge across tens and hundreds using partitioning
  Use straws, diennes, place value counters, coins, empty no lines, numicon.

Common mental calculation strategies:

- Partitioning and recombining
- Doubles and near doubles
- Use no pairs to 10 and 100
- Adding near multiples of 10 and adjusting
- Using patterns of similar calculations
- Use known number facts
- Bridging through ten, hundred
- Complimentary addition

Written calculations

Add numbers with up to three digits, using formal written (column) methods

Add to three digit numbers using physical and abstract representations (e.g. straws, diennes, place value counters, empty number lines, coins, numicon)

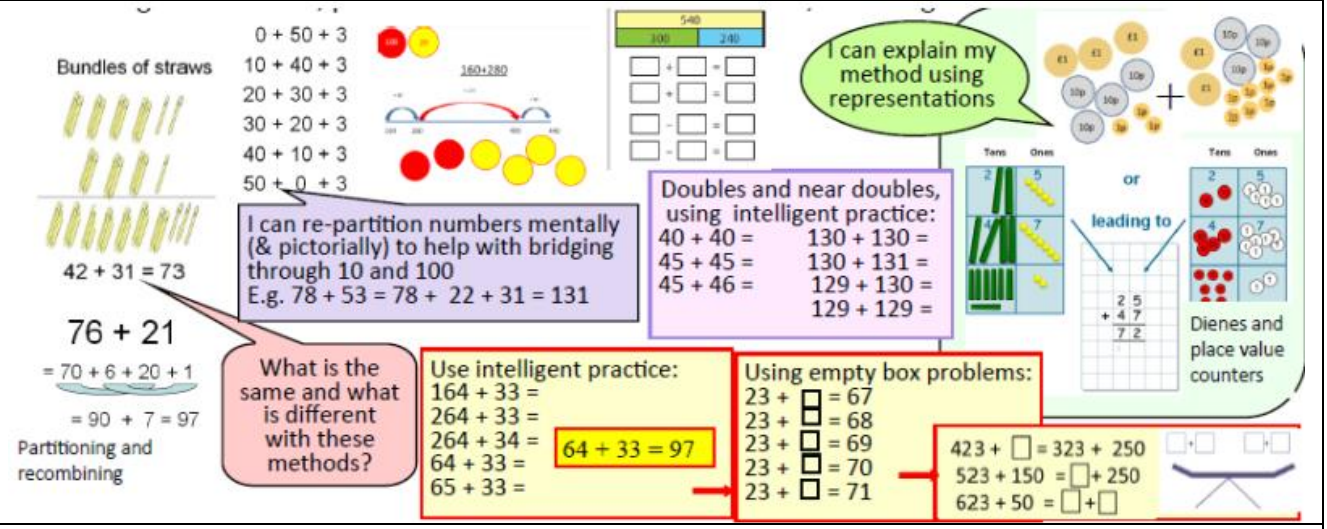
Use manipulatives to support structure of the algorithm especially place value

```
30 + 4          34
20 + 5    →    +25
-------         ---
50 + 9          59
```

```
200 + 30 + 4          234
500 + 20 + 7    →   + 527
------------          ---
700 + 60 + 1          761
    10                 1
```

NB. Informal methods of recording are used as stepping stones to help children understand the logic of formal written methods.

Representations to support



Fractions

Addition of fractions with the same denominator within one whole.

Mastery

Relating numbers to five and ten helps develop knowledge of the number bonds within 20. For example given 8 + 7, thinking of 7 as 2 + 5 and adding the 2 to 8 to make 10 and then the 5 to total 15. This should then be applied when calculating with larger numbers.

Year 4: Addition

Mental calculations

Practise mental methods with increasingly large numbers
Consolidate partitioning and re-partitioning
Bridge tens and hundreds using partitioning
Use compensation for adding too much/little and adjusting
Use straws, Diennes, numicon, place value counters, empty number lines etc.☐

Common mental calculation strategies:

- Partitioning and recombining
- Doubles and near doubles
- Use number pairs to 10 and 100
- Adding near multiples of ten and adjusting
- Using patterns of similar calculations
- Using known number facts
- Bridging though ten, hundred
- Complementary addition

Written calculations

Add numbers with up to four digits, using the formal written (column) method

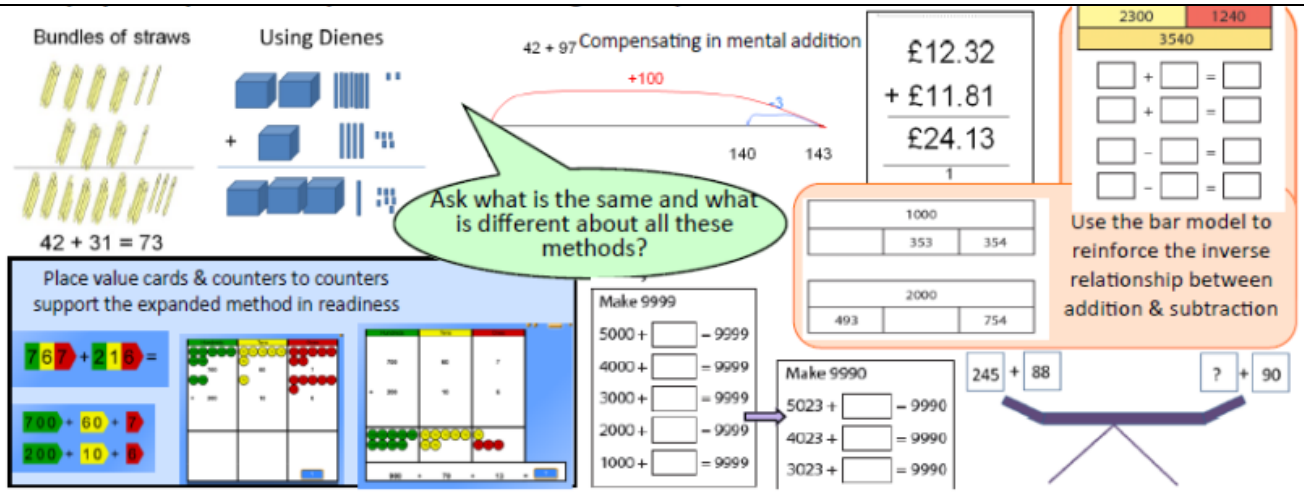
Add three digit numbers using column method and then move onto 4 digits.
Include decimal addition for money
Use manipulatives to support structure of the algorithm especially place value.

789 + 642 becomes

	7	8	9
+	6	4	2
1	4	3	1
	1	1	

Answer: 1431

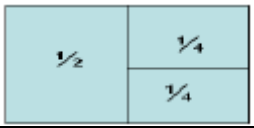
Representations to support



Fractions

Addition of fractions with the same denominator *to become fluent through a variety of increasingly complex problems beyond one whole*
Counting using simple fractions and decimals, both forwards and backwards

$$\frac{1}{2} + \frac{2}{4} = \frac{2}{4} + \frac{2}{4} = 1$$



Mastery

It helps to round numbers before carrying out a calculation to get a sense of the size of the answer. Looking at the numbers in a calculation and their relationship to each other can make calculating easier.

Year 5: Addition

Mental calculations

- **Add numbers mentally with increasingly large numbers, e.g. 12 462 + 2300 = 14 762**
- **Mentally add tenths, and one-digit numbers and tenths**
- **Add decimals, including a mix of whole numbers and decimals, decimals with different numbers of places, and complements of 1 (e.g. 0.83 + 0.17 = 1)**

Children use representation of choice
Refer back to pictorial and physical representations when needed.
Use concept of balance/equivalence to compensate
Bridge across boundaries by partitioning

Common mental calculation strategies:

- Partitioning and recombining
- Doubles and near doubles
- Use number pairs to 10 and 100
- Adding near multiples of ten and adjusting
- Using patterns of similar calculations
- Using known number facts
- Bridging though ten, hundred, tenth
- Complementary addition

Written calculations

Add whole numbers with more than four digits, using the formal written (column) method
Add three digit numbers using column method and then move onto 4 digits.
Include decimal addition for money
Use manipulatives to support structure of the algorithm especially place value

£563.14	24172m
+ £207.88	+ 5929m
£771.02	30101m
1 1 1	1 1 1 1

Representations to support

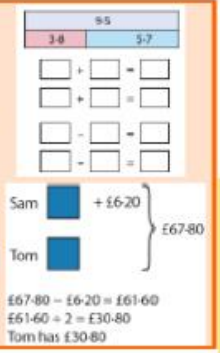
12 462 + 2300
= 12 462 + 2000 + 300
= 14 462 + 300
= 14 762

Partitioning and recombining

Jottings to support mental calculation

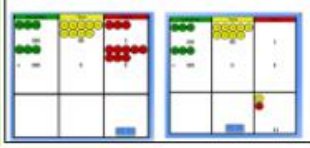
Use the **bar model** to reinforce the inverse relationship between addition & subtraction:

This supports problem solving: Sam and Tom have £67.80 between them. If Sam has £6.20 more than Tom, how much does Tom have?



1.6 + 1.4 = 3
Write down three more pairs of decimal numbers that sum to 3

Place Value counters to support column addition



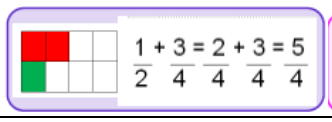
393
+ 308
1
1

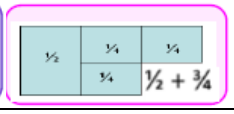
Compensating: true or false?
2741 + 1263 = 2742 + 1262
Why? Can you use resources or draw a picture to explain your answer? How can you adjust this to make the calculation easier? 3498 + 2067

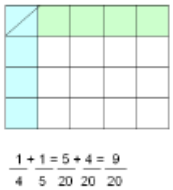
What is the same and **what is different** about all these methods?

Fractions

Add fractions with the same denominator and denominators that are multiples of the same number *(to become fluent through a variety of increasingly complex problems and add fractions that exceed 1 as a mixed number)*







Mastery

Before starting any calculation it is helpful to think about whether or not you can do it mentally.
Carrying out an equivalent calculation might be easier than the given calculation.

Year 6: Addition

Mental calculations

Perform mental calculations, including with mixed operations and large numbers
(more complex calculations)
Children use representation of choice
Consolidate partitioning and re-partitioning for bridging boundaries (tens, hundreds, thousands, tenths, hundredths …)
Use compensation for adding too much/little and adjusting
Refer back to pictorial and physical representations when needed.
Apply the rules of BIDMAS

Common mental calculation strategies:

- Partitioning and recombining
- Doubles and near doubles
- Use number pairs to 10 and 100
- Adding near multiples of ten and adjusting
- Using patterns of similar calculations
- Using known number facts
- Bridging though ten, hundred, tenth
- Complementary addition

Written calculations

Add larger numbers using the formal written (columnar) method
Add three digit numbers using columnar method and then move onto 4 digits.
Include decimal addition for money

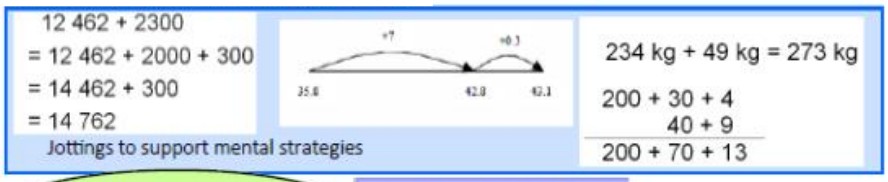
£563.14
+ £207.88
£771.02
1 1 1

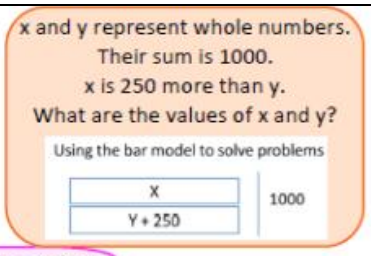
789 + 642 becomes

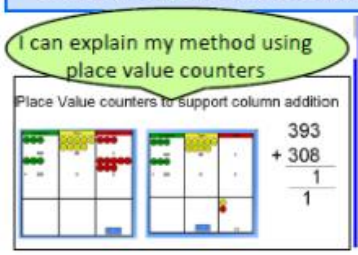
	7	8	9
+	6	4	2
1	4	3	1
	1	1	

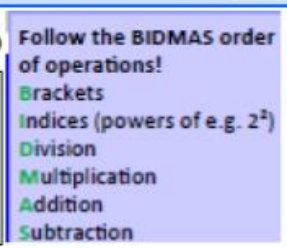
Answer: 1431

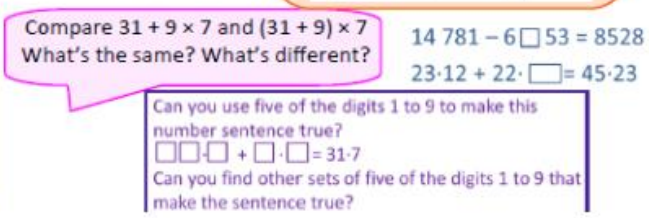
Representations to support











Fractions

Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions
Start with fractions where the denominator of one fraction is a multiple of the other (e.g. 1/2 + 1/8 = 5/8) and progress to varied and increasingly complex problems
Practise calculations with simple fractions and decimal equivalents to aid fluency

Mastery

Deciding which calculation method to use by being able to take apart and combine numbers in many ways e.g., calculating 8.78 + 5.26 might involve calculation 8.75 + 5.25 and then adjusting the answer. The associative rule helps when adding three or more numbers: 367 + 275 + 525 is probably best thought of as 367 + (275 + 525) rather than (367 + 275) + 525.

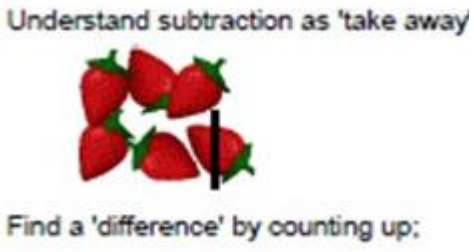
Early Years: Subtraction

Development Matters 40-60	• Recognise some numerals of personal significance. • Recognises numerals 1 to 5. • Counts up to three or four objects by saying one number name for each item. • Counts actions or objects which cannot be moved. • Counts objects to 10, and beginning to count beyond 10. • Counts out up to six objects from a larger group. • Selects the correct numeral to represent 1 to 5, then 1 to 10 objects. • Counts an irregular arrangement of up to ten objects. • Estimates how many objects they can see and checks by counting them. • Uses the language of 'more' and 'fewer' to compare two sets of objects. • Finds the total number of items in two groups by counting all of them. • Says the number that is one more than a given number. • Finds one more or one less from a group of up to five objects, then ten objects. • In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting. Records, using marks that they can interpret and explain. • Begins to identify own mathematical problems based on own interests and fascinations. **Early Learning Goal** Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.
Representations to support	 J[redacted] counted out 10 raisins. Then counted down as he ate them 10, 9, 8, 7, 6, 5, 4, 3, 2, 1. Hakima had chosen to collect all of the purple objects. She lined them all up and told me there were 16. She knew that 16 was a 1 and a 6. She could confidently tell me 1 and 2 more or less than 16, without counting, and she knew how to write each number.
Mastery	Children estimate a number of objects and check quantities by counting up to 20. They solve practical problems that involve combining groups of 2,5,10 or sharing into equal groups. Use these resources to support children in achieving 'Exceeding': https://nrich.maths.org/early-years

Year 1: Subtraction

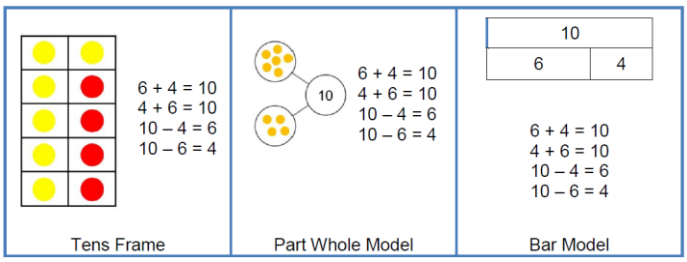
Mental calculations

- **Subtract one digit and two digit numbers to 20, including zero.**
- **Read, write and interpret mathematical statements using symbols (+ - =).**
- **Represent and use number bonds and related addition facts within 20.**
- **Solve one step problems using concrete objects and pictorial representations and missing number problems e.g., 9 - _ = 2.**
- Memorise and reason with number bonds.
- Subtract using numicon, cubes, number lines and tracks etc.
- Check with everyday objects
- Ensure pre-calculation steps are understood, including:
  - Counting objects
  - Conservation of number

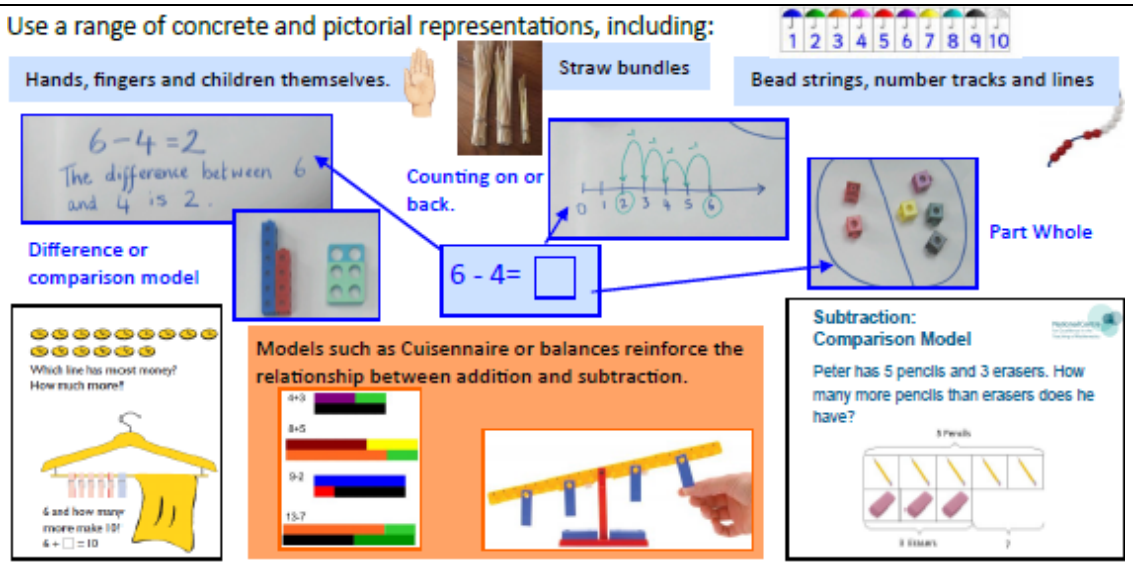


Written calculations

- **Subtract one digit and two digit numbers to 20.**
- **Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs.**
- **Represent and use number bonds and related addition facts within 20.**



Representations to support



Mastery

Relating numbers to five and ten helps develop knowledge of the number bonds within 20. For example given 8 + 7, thinking of 7 as 2 + 5 and adding the 2 to 8 to make 10 and then the 5 to total 15.

Thinking of part- whole relationships is helpful in linking addition and subtraction.

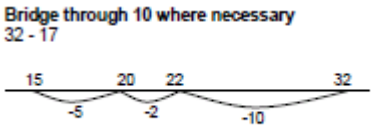
Year 2: Subtraction

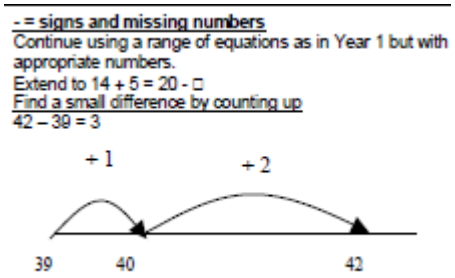
Mental calculations

- Add and subtraction numbers, using concrete objects, pictorial representations and mentally including:
  - A two-digit number and ones
  - A two-digit number and tens
  - Two two-digit numbers
  - Adding three one-digit numbers
- Jottings to support informal methods:

54—32 = 22

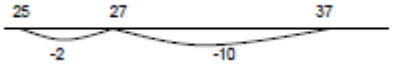
Written calculations



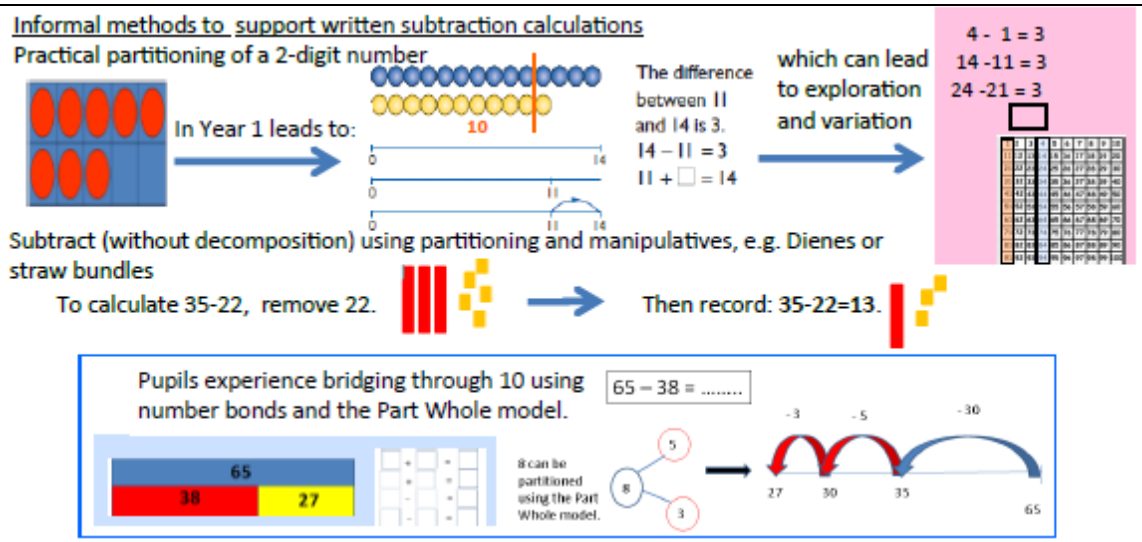


- Written recording:

37-12 = 37 - 10 – 2
= 27 – 2
= 25



Representations to support



Fractions

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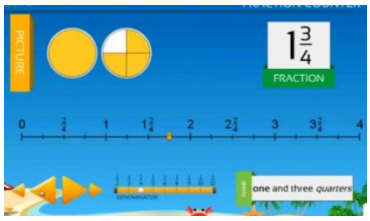


Mastery

Understanding that addition of two or more numbers can be done in any order.
When adding two numbers it can be more efficient to put the larger number first e.g., given 3 + 8 it is easier to calculate 8 + 3.
When adding three or more numbers it is helpful to look for pairs of numbers that are easy to add.
Understand the importance of the equal sign meaning 'equivalent' to is crucial for later work in algebra. Empty box problems can support the development of this key idea. Altering where the equals sign is placed develops fluency and flexibility.

Year 3: Subtraction

Mental calculations

- **Add and subtract numbers mentally including:**
  - **A three-digit number and ones**
  - **A three-digit number and tens**
  - **A three-digit number and hundreds**

247	
173	74

173 + 74 = 247

74 + 173 = 247

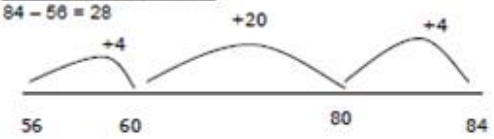
247 – 173 = 74

247 – 74 = 173

Use known number facts and place value to subtract
Continue as in Year 2 but with appropriate numbers e.g.
97 – 15 = 72

With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations
such as 57 – 12, 86 – 77 or 43 – 28.

Pencil and paper procedures
Complementary addition
84 – 56 = 28



Written calculations

Add and subtract numbers with up to three digits, using formal written (column) methods

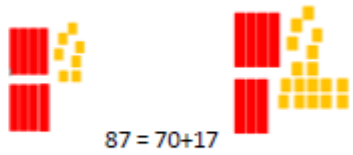
(1)Extended columnar - no exchange

Extended method 87 - 53 =

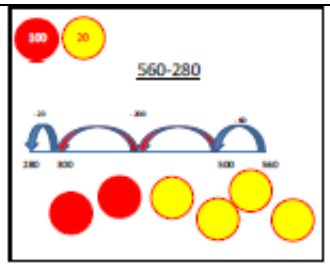
80 and 7
- 50 and 3
30 and 4 = 34

(2) Extended columnar – with exchange:
87-58 becomes

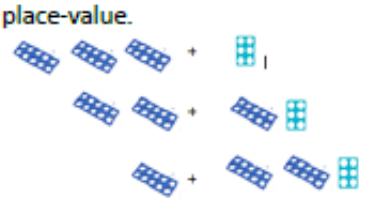
70 +1 7
-50 + 8
20 + 9



Representations to support



Partitioning and re- partitioning support the understanding of place-value.



30 + 6
20 + 16
10 + 26

All of these representations still comprise the amount of 36.

Introduce transition from concrete place value representations, (e.g. dienes or straws), to pictorial – such as place value counters or money.

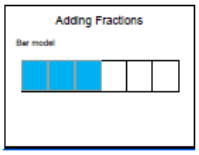
Revert to concrete manipulatives and expanded methods whenever difficulties arise

132 in Dienes 132 in place value counters.

Fractions

Count up and down in tenths.
Add and subtract numbers with the same denominator within one whole.

$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$



Mastery

Relating numbers to five and ten helps develop knowledge of the number bonds within 20. For example given 8 + 7, thinking of 7 as 2 + 5 and adding the 2 to 8 to make 10 and then the 5 to total 15. This should then be applied when calculating with larger numbers.
Subtraction bonds can be thought of in terms of addition: for example, in answering 15 – 8, thinking what needs to be added to 8 to make 15. Counting on for subtraction is a useful strategy that can also be applied to larger numbers.

Year 4: Subtraction

Mental calculations

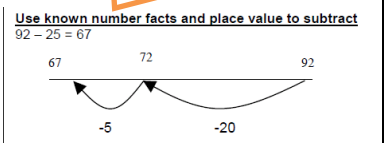
Continue to practise mental methods with increasingly large numbers

Methods to support fluent calculation and encourage efficiency of method:

- Find a small difference by counting up e.g. 5003 – 4996
- Subtract nearest multiple of ten and adjust
- Partition larger numbers

Whenever possible, children should be encouraged to visual number lines and other basic, supporting representations to promote fluent work without jottings.

This could be done using an empty number line. Children should recall and use number facts to reduce the number of steps.



Written calculations

Add and subtract numbers with up to four digits, using the formal written (column) method where possible.

Build on formal, extended method (*See Year 3)* using exchange wherever necessary.

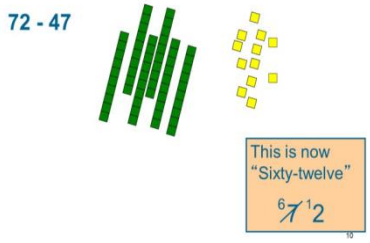
Continue to use representations and manipulatives to develop understanding of place value.

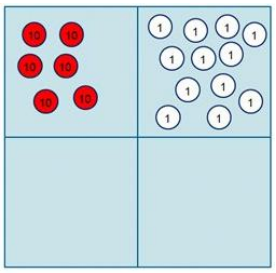
372—147 =

$$\begin{array}{r} 300 + 70 + 2 \\ -100 + 40 + 7 \end{array} \rightarrow \begin{array}{r} 300 + 60 + 12 \\ -100 + 40 + 7 \\ \hline 200 + 20 + 5 \end{array} \rightarrow \begin{array}{r} {}^{60}\not{70}\ {}^{1} \\ 300 + \quad + 2 \\ -\ 100 + 40 + 7 \\ \hline 200 + 20 + 5 \end{array}$$

Apply understanding of subtraction with larger integers to that of decimals in context of money and measures. (See Year 5.)

Representations to support





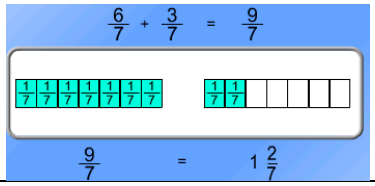
Diennes blocks or place value counters can be used to model calculations and the underlying place value of concepts.

$$\begin{array}{r} {}^{6}\not{7}\ {}^{1}2 \\ -\ 47 \\ \hline \end{array}$$

Use physical and/or pictorial representations and expanded algorithms alongside columnar methods. Ask: *What is the same/different?* Compare and discuss the suitability of different methods in context. Pupils **decide which operations and methods to use and why.**

I would count on using a number line to calculate 5002 – 4903 because the numbers are close together.

Fractions



Count up and down in hundreths.

Add and subtract fractions with the same demoniator.

Solve simple measures and money problems involving fractions and decimals to two decimal places.

Mastery

It helps to round numbers before carrying out a calculation to get a sense of the size of the answer. Looking at the numbers in a calculation and their relationship to each other can make calculating easier. Noticing that the numbers are close to each other might mean thinking about subtraction as difference.

Year 5: Subtraction

Mental calculations

- **Subtract numbers mentally with increasingly large numbers, e.g. 12,462 - 2300 = 10,126**
- **Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.**
- ***Pupils practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places and complements of 1 (e.g. 1 – 0.67 = 0.33)***
- ***Pupils mentally add and subtract tenths and one-digit whole numbers and tenths.***

Basic mental strategies for subtraction:

- Find difference by counting up
- Partitioning
- Applying known number facts
- Bridging through ten and multiples of ten
- Subtracting 9, 11 by compensating
- Counting on to or back from the larger number.

National Curriculum 1999

Which method works best? Why? How else could we do it?

ualise representation of choice.
Refer back to physical representations as required.

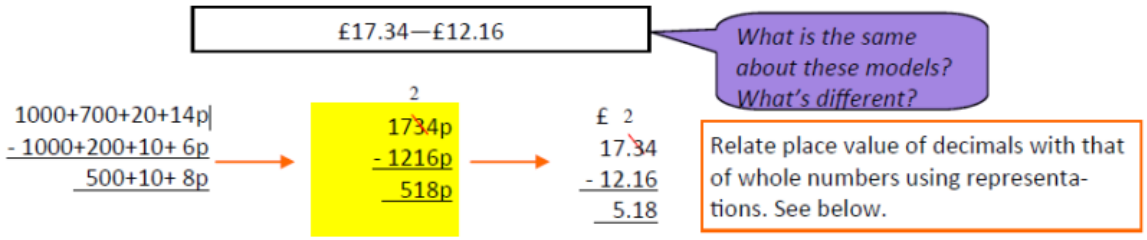
Written calculations

Add and subtract whole numbers with more than four digits, using the formal written (column) method

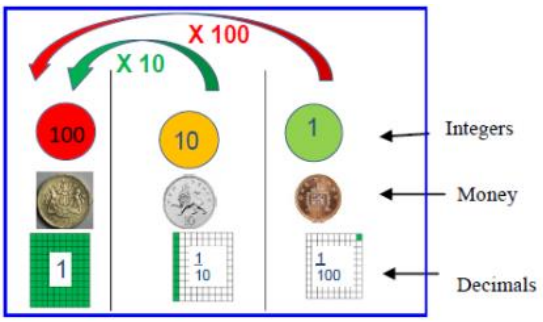
Practice adding and subtracting decimals.

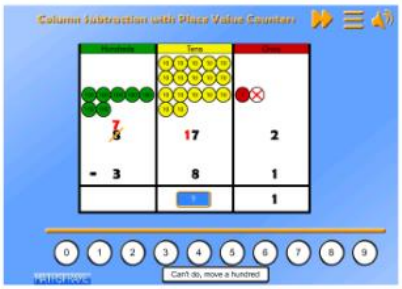
Begin with three-digit numbers using formal, columnar method, then move into four-digit numbers.

Use physical and/or pictorial representations and expanded algorithms alongside columnar methods. Ask: *What is the same/different?* Compare and discuss the suitability of different methods in context. Revert to expanded methods whenever difficulties arise.



Representations to support





Use physical and pictorial representations to stress the place value relationships between money, decimals and whole numbers. A place value mat such as this one could be used, moving away from the traditional *hundreds, tens and units* one used in KS1 and lower KS2.

Fractions

Subtract fractions with the same denominator and denominators that are multiples of the same number (*include fractions exceeding 1 as a mixed number*).

Solve problems involving numbers up to 3 decimal places.

Mentally add and subtract tenths and one-digit whole numbers and tenths.

Mastery

Before starting any calculation it is helpful to think about whether or not you can do it mentally. Carrying out an equivalent calculation might be easier than the given calculation.

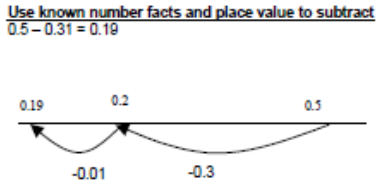
Year 6: Subtraction

Mental calculations

Children:

- **Perform mental calculations, including with mixed operations and large numbers**
- **Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.**
- ***They undertake mental calculations with increasingly larger numbers and more complex calculations.***

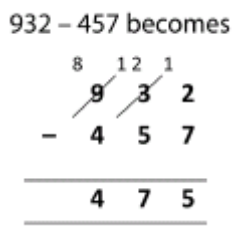
Children draw on basic, mental subtraction strategies (see Year 5). Children use or visualise representation of choice. Refer back to physical representations as required.



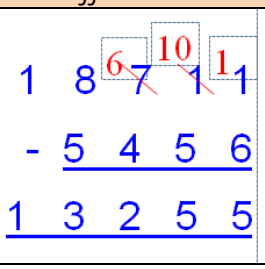
Written calculations

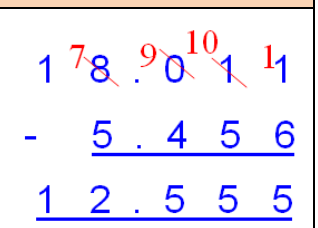
Add and subtract whole numbers with more than 4-digits using the formal written (columnar) method. Solve problems using the calculation and conversions of units of measure, using decimal notation of up to three decimal places where appropriate.

Move towards consolidation of formal, columnar method. For more complex calculations with increasingly larger or smaller numbers, compare representations and expanded algorithms alongside columnar methods. Ask "*what's the same/different?*" Compare and discuss the different methods (mental or written), in context. Revert to expanded methods where difficulties arise.

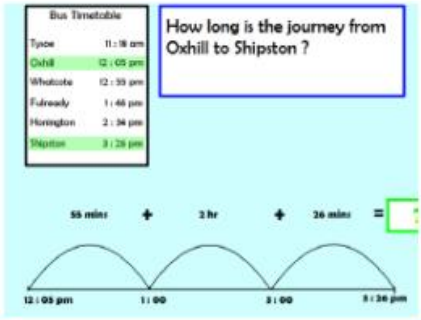


Consolidate columnar methods paying particular attention to the occurrence of zeroes as place holders.

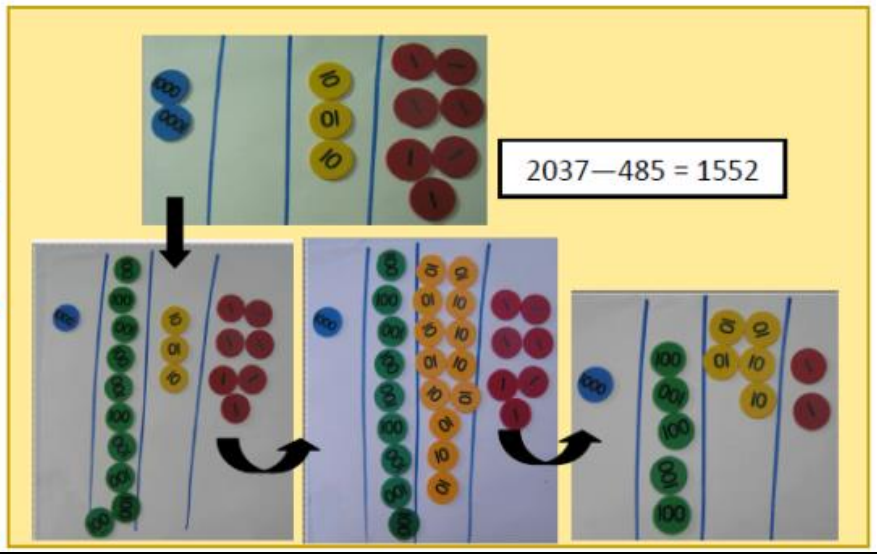




Representations to support



Use physical/pictorial representations alongside columnar methods where needed. *What is the same, what is different?*



Fractions

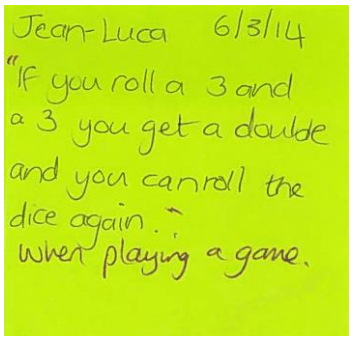
Add and subtract fractions with different denominators and mixed numbers.

Practise calculations with simple fractions and decimal fraction equivalents to aid fluency.

Mastery

Deciding which calculation method to use by being able to take apart and combine numbers in many ways e.g., calculating 8.78 + 5.26 might involve calculation 8.75 + 5.25 and then adjusting the answer. The associative rule helps when adding three or more numbers: 367 + 275 + 525 is probably best thought of as 367 + (275 + 525) rather than (367 + 275) + 525.

Early Years: Multiplication

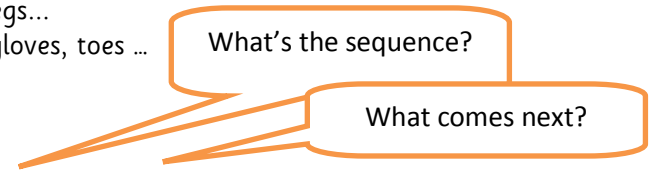
Development Matters 40-60	• Recognise some numerals of personal significance. • Recognises numerals 1 to 5. • Counts up to three or four objects by saying one number name for each item. • Counts actions or objects which cannot be moved. • Counts objects to 10, and beginning to count beyond 10. • Counts out up to six objects from a larger group. • Selects the correct numeral to represent 1 to 5, then 1 to 10 objects. • Counts an irregular arrangement of up to ten objects. • Estimates how many objects they can see and checks by counting them. • Uses the language of 'more' and 'fewer' to compare two sets of objects. • Finds the total number of items in two groups by counting all of them. • Says the number that is one more than a given number. • Finds one more or one less from a group of up to five objects, then ten objects. • In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting. Records, using marks that they can interpret and explain. • Begins to identify own mathematical problems based on own interests and fascinations. **Early Learning Goal** Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.
Representations to support	 C put two wheels on one side of his lorry. Now double it. He put two on the other side. "Thats four" 
Exceeding	Children estimate a number of objects and check quantities by counting up to 20. They solve practical problems that involve combining groups of 2,5,10 or sharing into equal groups. Use these resources to support children in achieving 'Exceeding': https://nrich.maths.org/early-years

Year 1: Multiplication

Mental calculations

- **Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**
- **Count in multiples of twos, fives and tens** with equipment, songs & rhythms, and including by rote

Counting 2s e.g. counting socks, shoes, animal legs…
Counting in 5 s e.g. counting fingers, fingers in gloves, toes …
Counting in 10s e.g. counting fingers, toes …
Doubles up to 10
Recognising odd and even numbers
Write as a number pattern (e.g. 5, 10, 15…; 2, 4, 6…; 10, 20, 30…)

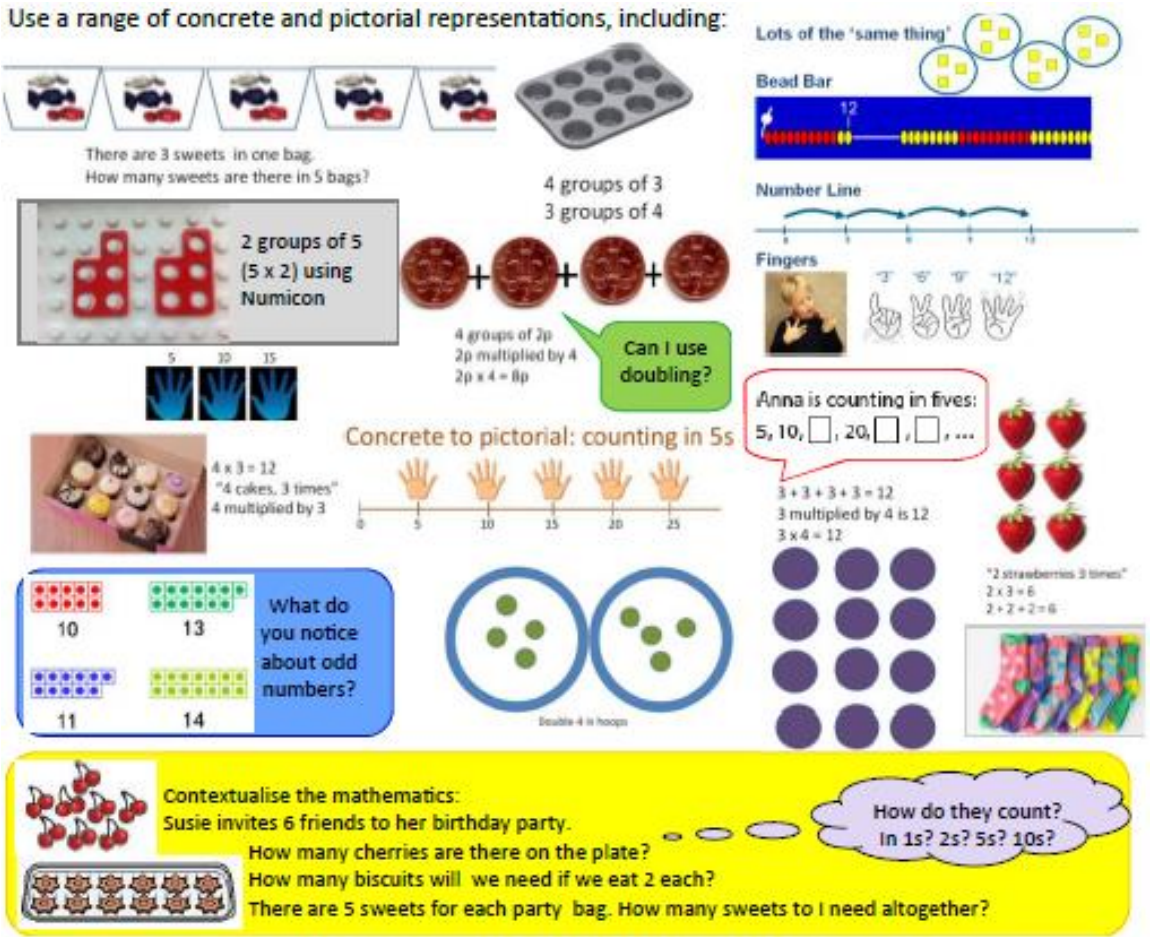


Written calculations

It is important to use a range of models to develop understanding of multiplication, and that children make connections between arrays, number patterns, and counting in twos, fives and tens.

Although there is no statutory requirement for written multiplication in year 1, it may be helpful to encourage children to begin to write it as a repeated addition number sentence in preparation for year 1. E.g. 2 + 2 + 2 + 2 = 8.

Representations to support



Mastery

Counting in steps of equal sized is based on the big idea of 'unitising' e.g. treating a group of 5 objects as one unit of 5.
Working with arrays helps pupils to become aware of the commutative property of multiplication, that 2 x 5 is equivalent to 5 x 2.

Year 2: Multiplication

Mental calculations

- **Recall and use multiplication and division facts for the 2,5,10 multiplication tables,** *connecting the 2,5,10 multiplication tables to each other.*
- Connect the 10 multiplcation table to place value.
- **Recognise odd and even numbers.**
- **Show that multiplication of two numbers can be done in any order (commutative).**
- *Use a variety of language to describe multiplication and division.*
- Applying doubling of numbers to 10 to doubling larger numbers.

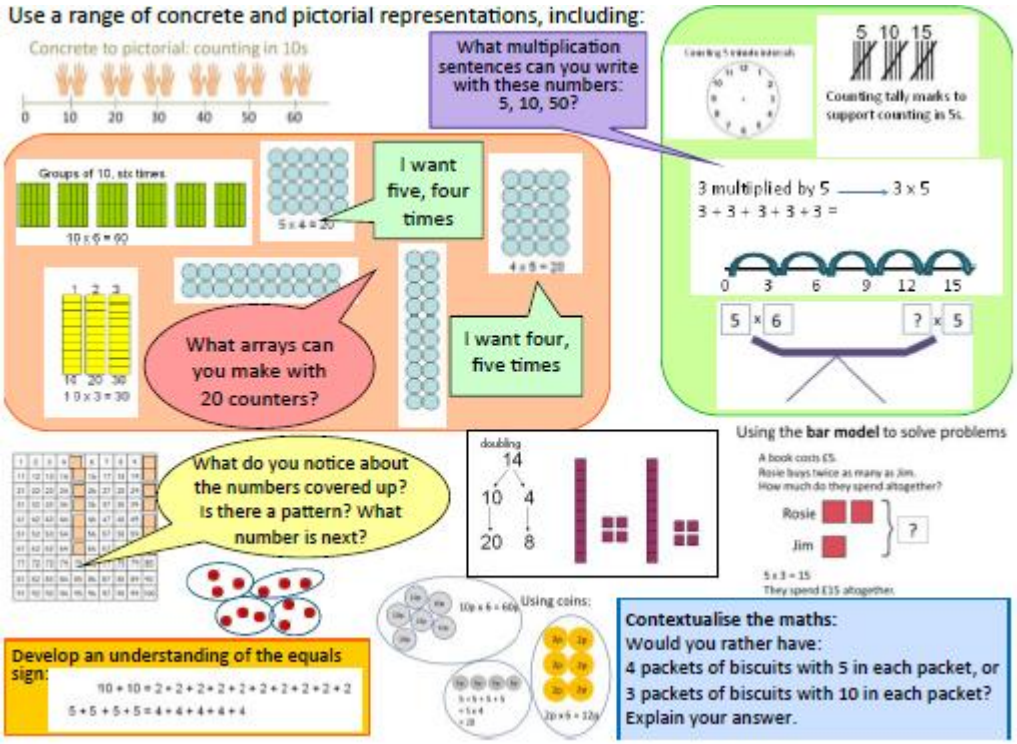
I know that multiples of 2,5,10 are always/never...

Written calculations

- **Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (X), division (÷) and equals (=) signs.**
- *Begin to use other multiplication tables and recall facts to perform written calculations.*
- Use a range of materials and contexts... including arrays and repeated addition.

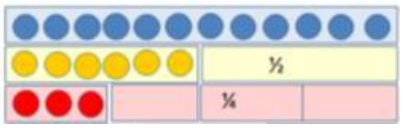
7 x 2 = □
7 x □ = 14
□ x 2 = 14
△ x □ = 14

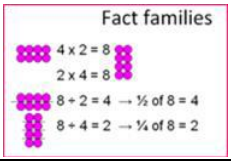
Representations to support



Fractions

- **Write simple fractions e.g. ½ of 6 = 3 and recognise the equivalence of two quarters and one half.**
- *Begin to relate multiplication and division models to fractions and measures.*





Mastery

It is important that pupils commit both multiplication facts to memory and also develop an understanding of conceptual relationships. This will aid them in using known facts to work out unknown facts and in solving problems.

Pupils should look for and recognise patterns within tables and connections between them (e.g. 5x is half 10x).

Pupils should recognise multiplication and division as inverse operations and use this knowledge to solve problems.

The recognition of pattern in multiplication helps pupils commit facts to memory, for example doubling twice is the same as multiplying by four.

Year 3: Multiplication

Mental calculations

- **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** (and 2, 5 and 10 multiplication tables from Y2)
- Use doubling to connect 2, 4 and 8 multiplication tables
- Develop efficient mental methods using commutativity and associativity
- Derive related multiplication and division facts
- **Calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods**
- Partitioning: multiply the tens first and then multiply the units,
  e.g. 57 x 6 = (50 x 6) + (7 x 6) = 300 + 42 = 342
- Children can apply these skills to solve spoken word problems too,
- Include missing number statements e.g. 72 ÷ _ = 8

The associative law:
4 x 12 x 5 = 4 x 5 12
= 20 x 12
= 240

The commutative law:
4 x 12 = 12 x 4

I have 8 packets, each containing 12 crayons. How many crayons do I have in total?

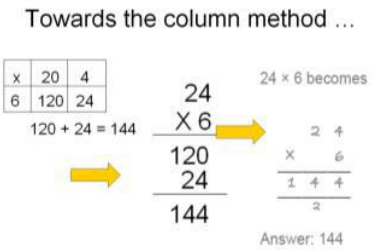
Ensure opportunities to learn multiplication tables through use of visual models, images and also rote learning.

Multiplication and division facts:
8 x 4 = 32, 4 x 8 = 32, 32 ÷ 4 = 8, 32 ÷ 8 = 4

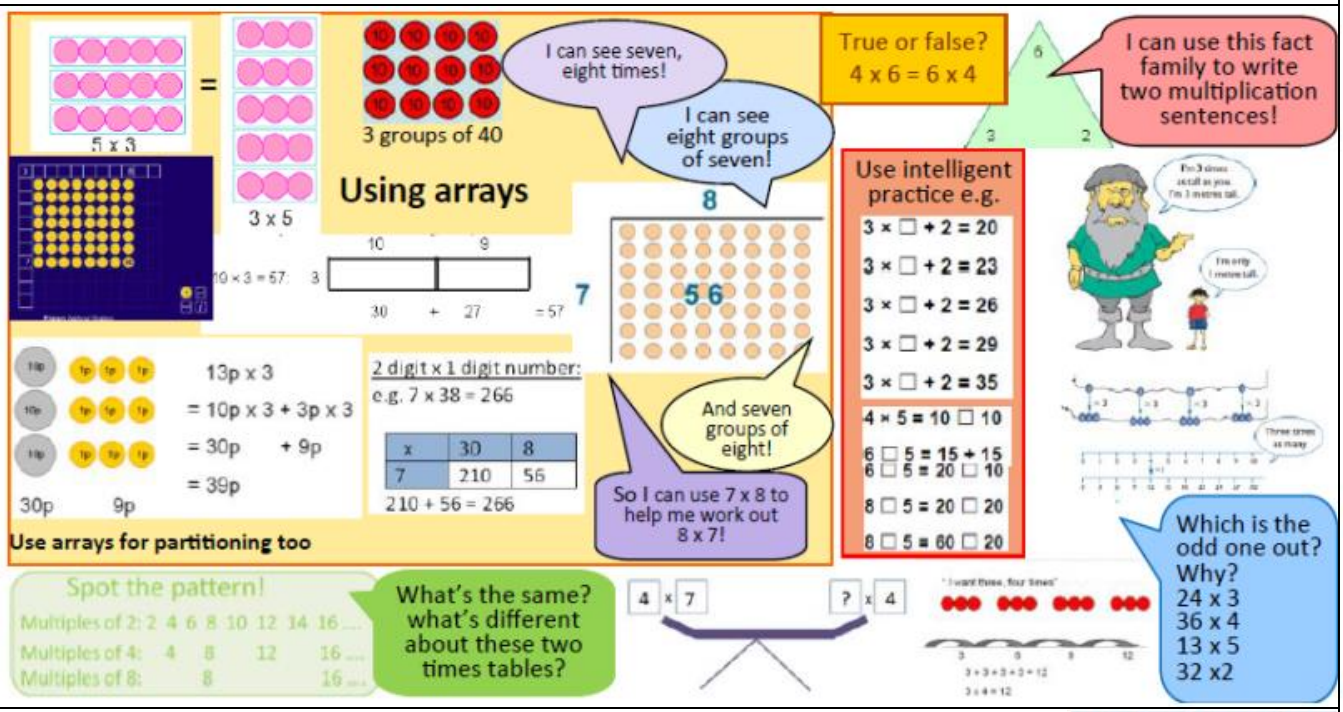
Deriving related facts:
3 x 2 = 60, 6 ÷ 3 = 2, 6 ÷ 2 = 3
➡ 30 x 2 = 60, 60 ÷ 3 = 20, 20 = 60 ÷ 3

Written calculations

- **Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods**
- Estimate before calculating
- Ensure written methods build on/relate to mental methods

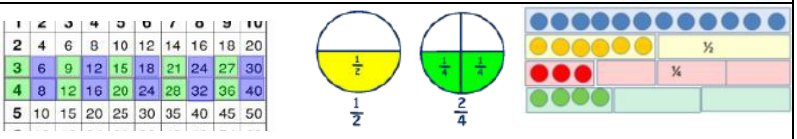


Representations to support



Fractions

Recognise and show, using diagrams, equivalent fractions with small denominators.



Mastery	It is important for children not just to be able to chant their multiplication tables but also to understand what the facts in them mean, to be able to use these facts to figure out others and to use in problems. It is also important for children to be able to link facts within the tables (e.g. 5X is half 10X). They understand what multiplication means, see division as both grouping and sharing and see division as the inverse of multiplication.

Year 4: Multiplication

Mental calculations

- **Recall multiplication and division facts for multiplication tables up to 12 x 12**
- **Use place value, known and derived facts to multiply and divide mentally including:**
  - **Multiplying by 0 and 1**
  - **Dividing by 1**
  - **Multiplying together 3 numbers.**
- **Recognise and use factor pairs and commutivity in mental calculations**
- Practise mental methods and extend this to three-digit numbers to derive facts (e.g. 600 ÷ 3 = 200 can be derived from 6 ÷ 3 = 2)
- Apply understanding of the equals sign
- Link facts within the tables (e.g. 5X is half 10X)

Using the **distributive** law:
39 x 7 = 30 x 7 + 9 x 7

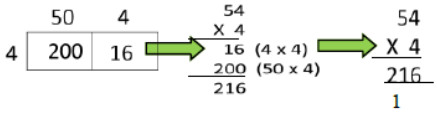
Using the **associative** law:
(2 x 3) x 4 = 2 x (3 x 4)

Using facts and rules:

2 x 6 x 5 = 10 x 6 = 60

Written calculations

- **multiply two-digit and three-digit numbers by a one-digit number using formal written layout**
- Estimate before calculating
- Ensure written methods build on/relate to mental methods (e.g. grid method) based on an understanding of place value
- Use grid and expanded column methods as stepping stones alongside



Key skills to support:

- Know or quickly recall multiplication facts up to 12 x 12
- Under the effect of multiplying numbers by 10, 100 or 1000
- Multiply multiples of 10, e.g. 20 x 40
- Approximate e.g. recognise that 72 x 38 is approximately 70 x 40 = 2800 and use this information to check whether their answer appears sensible.

Revert to expanded methods if children find formal calculation method difficult.

Representations to support

Ensure children can confidently multiply & divide by 10 and 100, that multiplying by 10 makes the number bigger and all digits move one place to the left, while dividing by 10 makes the number smaller and all the digits move one place to the right.

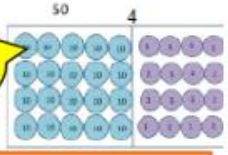
Moving digits ITP

245
X 6
1470
23

This digit is worth 200

This digit is worth 30

I can use place value counters to model the grid method



Use intelligent practice e.g.

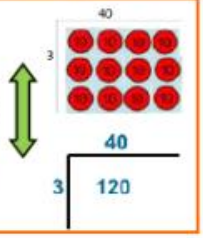
2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

Three ways to calculate 7 x 6:

7 x 6 = 7 x 5 + ☐ 7 x 6 = 7 x 7 - ☐ 7 x 6 = ☐ x ☐

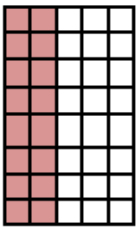
Now find the answer to 6 x 9 in three different ways.

Use arrays made with place value counters to demonstrate the link between multiplication and division. This will support understanding of the grid method.



Children need to understand and apply the language of multiples and factors and use it in solving multiplication and division problems e.g.
'All factors of 36 are multiples of 2, true or false?
Find me two factors of 48 that are also multiples of 3.'

Using the bar model to solve problems:

Fractions	• **Recognise and show, using diagrams, families of common equivalent fractions** • Understand the relation between non unit fractions and multiplicationand divisuon of quantities, with particular emphasis on tenths and hundreths • Make connections between fractions of a length, of a shape and as a representation of one qhole or set of quantities $\frac{4}{10}\ \frac{6}{15}\ \frac{8}{20}\ \frac{10}{25}\ \frac{12}{30}\ \frac{14}{35}\ \frac{16}{40}$ $\frac{2}{5} = \frac{16}{40}$  • Use factors and mutliples to recognise equivalent fractions and simplify where appropriate.
Mastery	It is important for children not just to be able to chant their multiplication tables but also to understand what the facts in them mean, to be able to use these facts to figure out others and to use in problems. It is also important for children to be able to link facts within the tables (e.g. 5X is half 10X). They understand what multiplication means, see division as both grouping and sharing and see division as the inverse of multiplication. The distributive law can be used to partition numbers in different ways to create equivalent calculations. Looking for equivalent calculations can make calculating easier. The array model can help show equivalences.

Year 5: Multiplication

Mental calculations

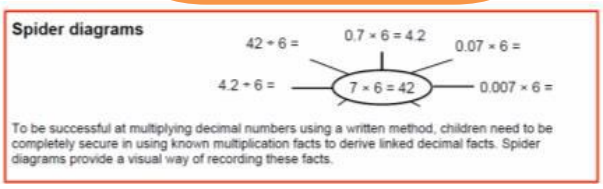
- Multiply and divide numbers mentally drawing upon known facts.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Recognise and use square and cube numbers (& notation).
- Use factors and multiplier as connected ideas: 48 is a multiple of 6 and 6 is a factor of 48.

Pupils should be taught throughout that decimals, percentages and fractions are different ways of expressing proportions.

24 x 15 =

I did: 24 x 5 = 120 (half of 24 x 10), then multiplied 120 by 3 to get 360.

I did: (24 x 10) + (24 x 5).



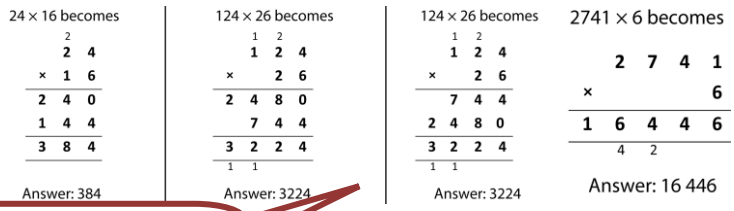
Examples of constructing equivalence statements:

4 x 35 = 2 x 2 x 35

3 x 270 = 3 x 3 x 9 x 10 = 92 x 10

Written calculations

- **Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.**

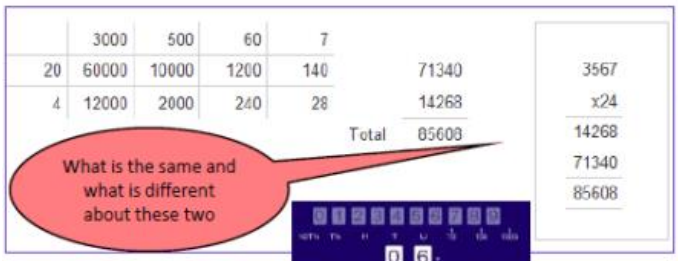


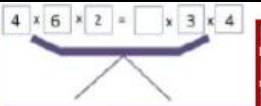
Does your answer seem reasonable?

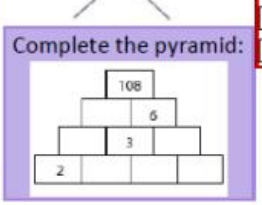
Compact methods for multiplication are efficient but often do not make the value of each digit explicit. When introducing multiplication of decimals, it is sensible to take children back to an expanded form such as the grid method where the value of each digit is clear, to ensure that children understand the process.

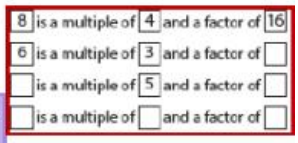
Revert to expanded methods if children find formal calculation method difficult. (year 3/4)

Representations to support



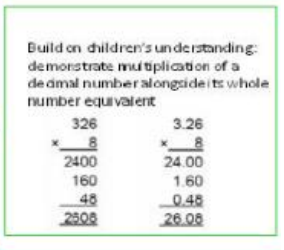


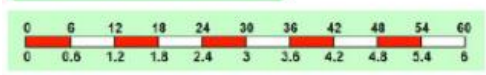


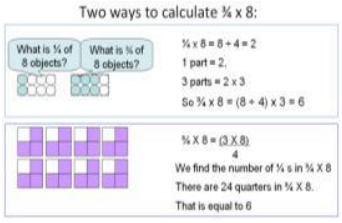
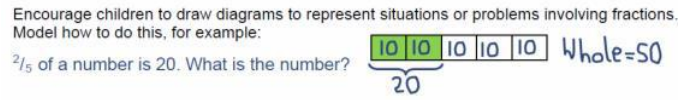


12 x 6 = 24 x 3
True or false? Prove it!

Start multiplying by using the **least significant digit** for the grid method will support children with implementation of the written procedure





Fractions	• **Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams** • **Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths** • Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, building on work from previous years. This relates to scaling by simple fractions, including fractions > 1.  
Mastery	Pupils have a firm understanding of what multiplication and division mean and have a range of strategies for dealing with large numbers, including both mental and standard written methods. They see the idea of factors, multiples and prime numbers as connected and not separate ideas to learn. They recognise how to use their skills of multiplying and dividing in new problem solving situations. Fractions and division are connected ideas: 36 ÷ 18 = 36/18 = 2; 18/36 = ½ Factors and multiples are connected ideas: 48 is a multiple of 6 and 6 is a factor of 48.

Year 6: Multiplication

Mental calculations

- **Perform mental calculations, including with mixed operations and large numbers** *(increasingly large numbers and more complex calculations)*
- *Use all the multiplication tables to calculate mathematical statements in order to maintain fluency.*
- **Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.**
- **Identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places.**
- Use and apply connections between factors, multiples and prime numbers and between fractions, division and ratios.

Use mental strategies to solve problems e.g.
- x4 by doubling and doubling again
- x5 by x10 and halving
- x20 by x10 and doubling
- x9 by multiplying by 10 and adjusting
- x6 by multiplying by 3 and doubling

What is the best approximation for 4.4 x 18.6?

Children should know the square numbers up to 12 × 12 & derive the corresponding squares of multiples of 10 e.g. 80 × 80 = 6400

How many different x/÷ facts can you make for 72? 7.2? 0.72?

Written calculations

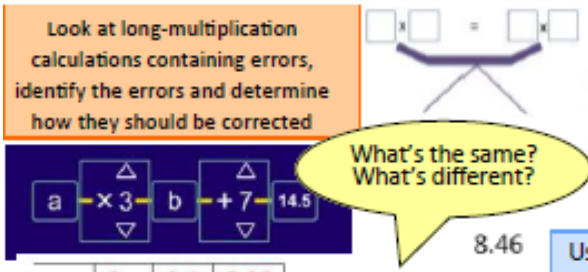
- **Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication** *(short & long multiplication)*
- **Multiply one-digit numbers with up to two decimal places by whole numbers**
- Understand that standard written multiplication method involves a number of partial products e.g. 36 × 24 is made up of four partial products 30 × 20, 30 × 4, 6 × 20, 6 × 4.
- Use manipulatives to support structure of the algorithm especially place value.

£	6.23
x	27
	43.61
	124.60
£	168.21

Revert to expanded methods if children find formal calculation method difficult. (year 4/5)

Representations to support

Look at long-multiplication calculations containing errors, identify the errors and determine how they should be corrected



What's the same? What's different?

x	8	0.4	0.06
11	88	4.4	0.66

= 93.06

8.46 X 11 = 93.06

Using the bar model to solve problems:

A gardener plants tulip bulbs in a flower bed. She plats 3 red bulbs for every 4 white bulbs. She plants 60 red bulbs. How many white bulbs does she plant?

Use questioning to develop conceptual understanding e.g.
Which is the odd one out?
24 x 3 36 x 4 13 x 5 32 x 2

Use empty box questions:

□ × □ = 864

□ × □ × □ = 864

BODMAS
- 8·4 x 3 + 8·4 x 7
- 6·7 x 5 − 0·67 x 50
- 93 x 0·2 + 0·8 x 93
- 7·2 x 4 + 3·6 x 8

Fractions

- Multiply simple pairs of proper fractions, writing the answer in its simplest form, e.g. ¼ x ½ = 1/8.

Three key applications of understanding:
- Recognise that ¼ of 12, ¼ x 12 and 12 divided by 4 are equivalent
- Use cancellation to simplify the product of a fraction and an integer, e.g. 1/5 of 15 = 3, 2/5 x15 = 2 x 1/5 x 15 = 2 x 3 = 6
- Work out how many 1/2s in 15, how many 2/5s in 15, how many 2/5s in 1 etc.

To calculate ¼ x ½ , find ½ of a rectangle/array, then divide that ½ into ¼s. So ¼ of ½ is 1/8

Pupils should use a variety of images to support their understanding of multiplication with fractions. This follows earlier work about fractions as operators (fractions of), as numbers and as equal parts of objects, e.g. as parts of a rectangle.

Mastery	Standard written algorithms use the conceptual structures of the mathematics to produce efficient methods of calculation. Standard written multiplication method involved a number of partial products, e.g. 36 x 24 is made up of four partial products 30 x 20, 30 x 4, 6 x 20, 6 x 4. There are connections between factors, multiples and prime numbers and between fractions, division and ratios.

Early Years: Division

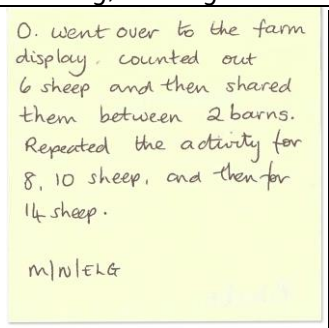
Development Matters 40-60

- Recognise some numerals of personal significance.
- Recognises numerals 1 to 5.
- Counts up to three or four objects by saying one number name for each item.
- Counts actions or objects which cannot be moved.
- Counts objects to 10, and beginning to count beyond 10.
- Counts out up to six objects from a larger group.
- Selects the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Counts an irregular arrangement of up to ten objects.
- Estimates how many objects they can see and checks by counting them.
- Uses the language of 'more' and 'fewer' to compare two sets of objects.
- Finds the total number of items in two groups by counting all of them.
- Says the number that is one more than a given number.
- Finds one more or one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting. Records, using marks that they can interpret and explain.
- Begins to identify own mathematical problems based on own interests and fascinations.

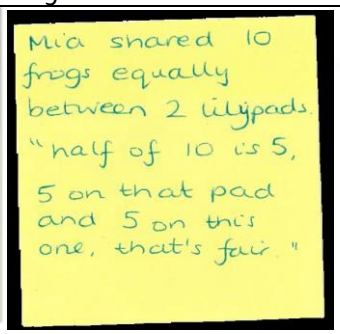
Early Learning Goal

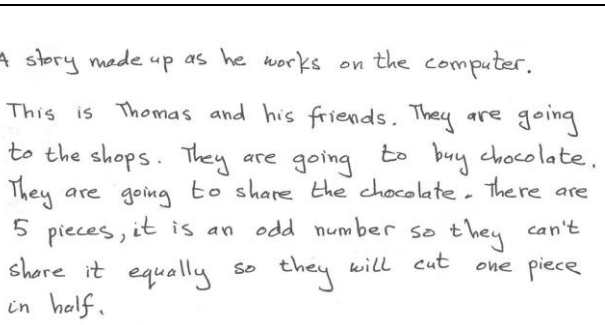
Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Representations to support


O. went over to the farm display. counted out 6 sheep and then shared them between 2 barns. Repeated the activity for 8, 10 sheep, and then for 14 sheep.

M|N|ELG


Mia shared 10 frogs equally between 2 lilypads. "half of 10 is 5, 5 on that pad and 5 on this one, that's fair."


A story made up as he works on the computer.

This is Thomas and his friends. They are going to the shops. They are going to buy chocolate. They are going to share the chocolate. There are 5 pieces, it is an odd number so they can't share it equally so they will cut one piece in half.

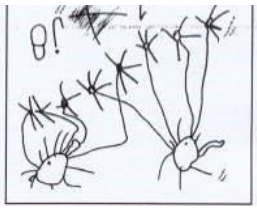
Exceeding

Children estimate a number of objects and check quantities by counting up to 20. They solve practical problems that involve combining groups of 2,5,10 or sharing into equal groups.
Use these resources to support children in achieving 'Exceeding':
https://nrich.maths.org/early-years

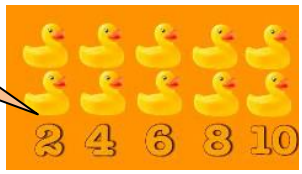
Year 1: Division

Mental calculations

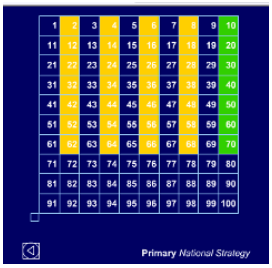
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
(Pupils) make connections between arrays, number patterns, and counting in twos, fives and tens.



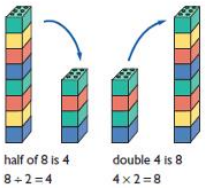
Songs are useful for counting in steps



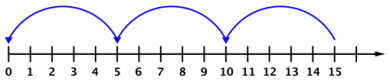
Count on or back in 2s, 5s and 10s and look for patterns.



Written calculations



Children should experiment with the concepts of sharing and grouping in a number of contexts. Initially they use their own recording—moving towards fluent, symbolic notation in Year 2. Conceptual understanding and recording should be continuously supported by the use of **arrays** as a default model, as well as other representations, (see below.)



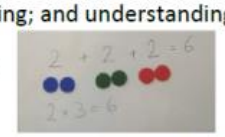
The relationship between multiplication and division must be continually considered.

Representations to support

Use a range of concrete and pictorial representations, including:

- Manipulatives to support children's own recording; and understanding of *sharing* and the link with multiplication.
  "How can we share 6 cakes between 2 people?"

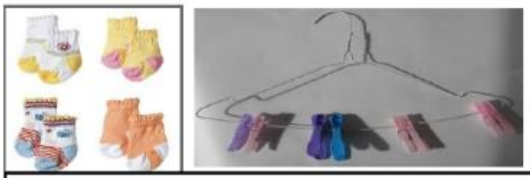
Here, the cakes are placed in an array formation.



How many 2 tiles can we fit on the 6 tile?

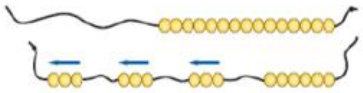
Moving from concrete to pictorial, counters represent the cakes to reinforce the relationship between multiplication and division.

- Manipulatives, and real-life objects to support children's own recording; and understanding of *grouping* and the link with multiplication.



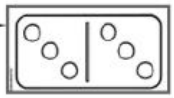
Coat hangers and socks support calculation of 8÷2

Bead strings



15 ÷ 2 using grouping model

"Double 3 is 6. Half of 6 is 3."



- Dominoes and dice to reinforce concepts of doubling and halving.

Fractions

Recognise, find and name a half as one of two equal parts of an object, shape or quantity
Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.
(*See* Representations *above.)*

Mastery

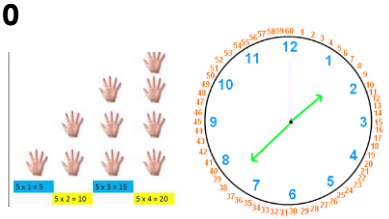
Counting in steps of equal sized is based on the big idea of 'unitising' e.g. treating a group of 5 objects as one unit of 5.
Working with arrays helps pupils to become aware of the commutative property of multiplication, that 2 x 5 is equivalent to 5 x 2.

Year 2: Division

Mental calculations

The relationship between multiplication and division must be continually considered.

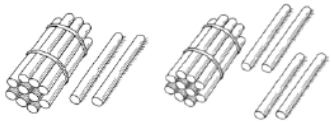
- **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers .**
- **Calculate mathematical statements for multiplication and division within the multiplication tables and write them**
- **Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot**



Written calculations

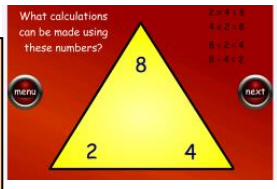
- **Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. (*See below.)***

"There are 26 straws. ½ of the straws is equal to 13"



½ of 26 = 13
26 ÷ 2 = 13

Pupils decode a problem first, represent it using manipulatives and jottings and finally record it symbolically.



Representations to support

Use a range of concrete and pictorial representations, including:

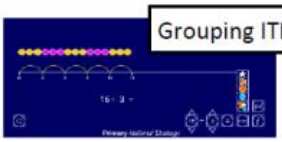
- Arrays

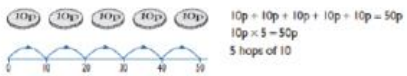
7 x 2 = 14
14 ÷ 2 = 7

2 x 7 = 14
14 ÷ 7 = 2

Is 14 an odd number? How do you know?

- Number lines to support grouping





"How many groups of 5 minutes have passed when the minute hand reaches twenty past?"

- Representations to support multiplicative reasoning:

Using Dienes: "If 40 ÷ 10 = 4 and 30 ÷ 10 = 3, what do you think 70 ÷ 10 would be? Why?"

Fractions

Recognise, find, name and write fractions ⅓, ¼, ¾, 2/4 of a length, shape, set of objects or quantity
Write simple fractions for example, ½ of 6 = 3 and recognise the equivalence of ½ and 2/4.

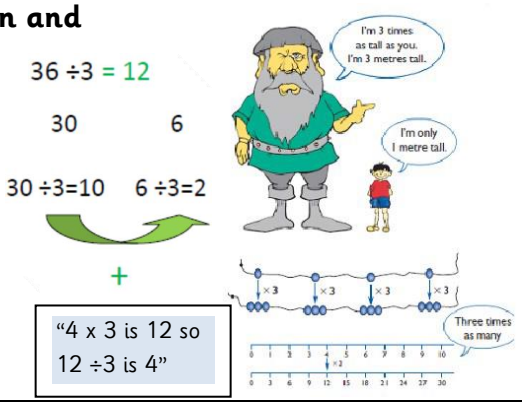
Mastery

It is important that pupils commit both multiplication facts to memory and also develop an understanding of conceptual relationships. This will aid them in using known facts to work out unknown facts and in solving problems.
Pupils should look for and recognise patterns within tables and connections between them (e.g. 5x is half 10x).
Pupils should recognise multiplication and division as inverse operations and use this knowledge to solve problems. They should also recognise division as both grouping and sharing.
The recognition of pattern in multiplication helps pupils commit facts to memory, for example doubling twice is the same as multiplying by four.

Year 3: Division

Mental calculations

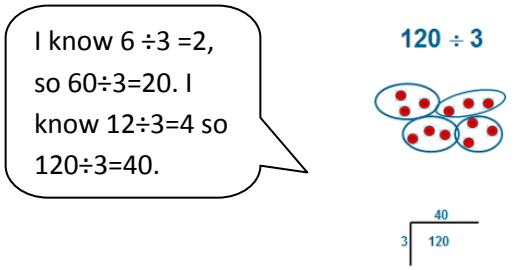
Pupils should be taught to recall and use multiplication and division facts for the 3,4, and 8 multiplication tables.
Pupils continue to practise their mental recall of multiplication tables… in order to improve fluency.
Pupils develop efficient mental methods, e.g using commutativity and associativity
(e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240) and multiplication and division facts to derive related facts.



Written calculations

Pupils should be taught to:

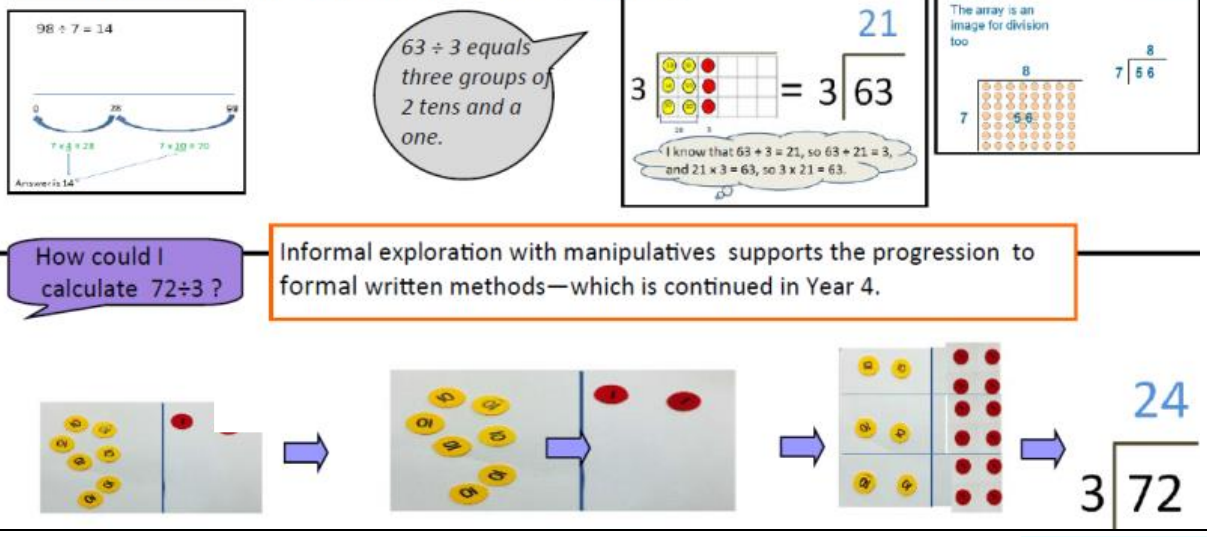
- **Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers time one-digit numbers, using mental and progressing to formal written methods.**
- **Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.**



New written methods can be modelled alongside mental or informal methods to ensure understanding.

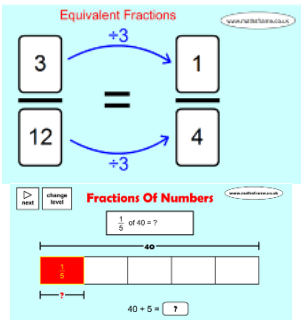
Representations to support

Use a range of concrete and pictorial resources, including:



Fractions

- **Recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10.**
- **Recognise and show, using diagrams, equivalent fractions with small denominators.**
- **Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.**



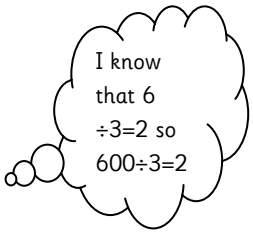
Mastery	It is important for children not just to be able to chant their multiplication tables but also to understand what the facts in them mean, to be able to use these facts to figure out others and to use in problems. It is also important for children to be able to link facts within the tables (e.g. 5X is half 10X). They understand what multiplication means, see division as both grouping and sharing and see division as the inverse of multiplication.

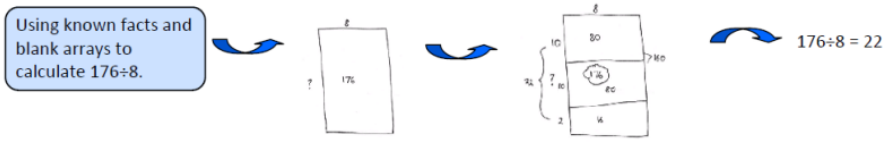
Year 4: Division

Mental calculations

Pupils should be taught to:

- **Recall multiplication and division facts for multiplication tables up to 12 x 12**
- **Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1, dividing by 1, multiplying together three numbers**
- **Recognise and use factor pairs and commutativity in mental calculations.**





Pupils practise mental methods and extend this to three-digit numbers to derive facts.

Written calculations

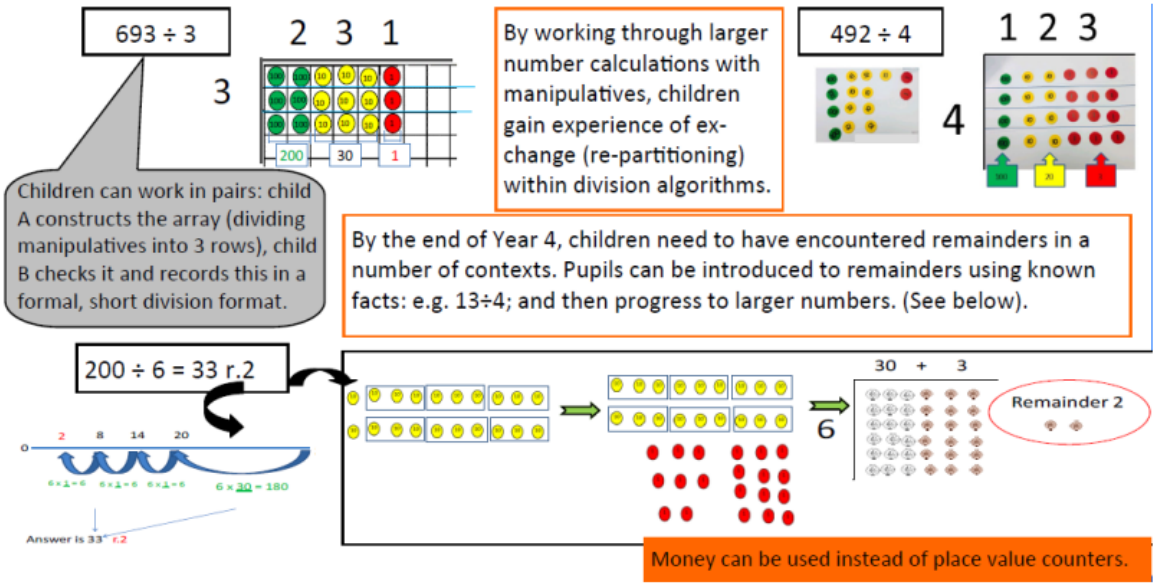
Pupils should be taught to:

- **multiply two-digit and three-digit numbers by a one-digit number using formal written layout**
- **solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.**

Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers.

Revert to expanded methods if children find formal calculation method difficult.

Representations to support



Fractions

Pupils should be taught to:

- ***Recognise and show, using diagrams, families of common equivalent fractions***
- ***Recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.***
- ***Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number***
- ***Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths***

Mastery	It is important for children not just to be able to chant their multiplication tables but also to understand what the facts in them mean, to be able to use these facts to figure out others and to use in problems. It is also important for children to be able to link facts within the tables (e.g. 5X is half 10X). They understand what multiplication means, see division as both grouping and sharing and see division as the inverse of multiplication. The distributive law can be used to partition numbers in different ways to create equivalent calculations. Looking for equivalent calculations can make calculating easier. The array model can help show equivalences.

Year 5: Division

Mental calculations

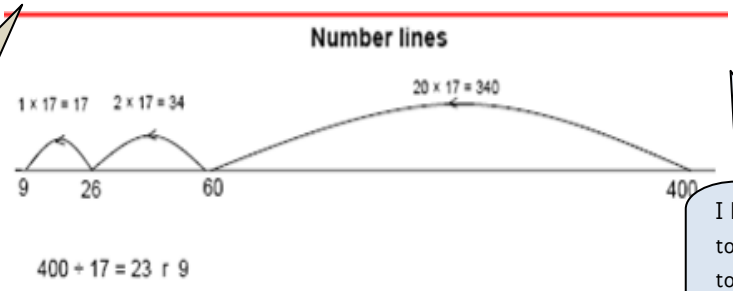
Pupils should be taught to:

- **multiply and divide whole numbers and those involving decimals by 10, 100 and 1000**
- **multiply and divide numbers mentally drawing upon known facts**
- **identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers .**

If 42 ÷ 6 = 7

÷ 10 ÷10

Then 4.2 ÷ 6 = 0.7



400 ÷ 17 = 23 r 9

Factorising

480 ÷ 15

= 480 ÷ 5 ÷ 3

I know that the answer to 138 ÷ 6 will be close to 20 because 2 x 6 = 12 so 20 x 6 = 120.

Pupils apply all the multiplication tables and related division facts frequently and use them confidently.

Written calculations

Pupils practise and extend their use of the formal written methods of short multiplication and short division.

- **Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**

98 ÷ 7 becomes

$$\begin{array}{r} 1\ 4 \\ 7\overline{)\,9^{2}\,8} \end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r} 8\ 6\ r2 \\ 5\overline{)\,4\,3^{3}\,2} \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r} 4\ 5\ r1 \\ 1\ 1\overline{)\,4\,9^{5}\,6} \end{array}$$

Answer: $45\frac{1}{11}$

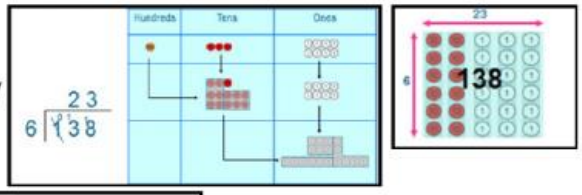
- **Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding. (See Representations below.)**

Revert to expanded methods if children find formal calculation method difficult.

Representations to support

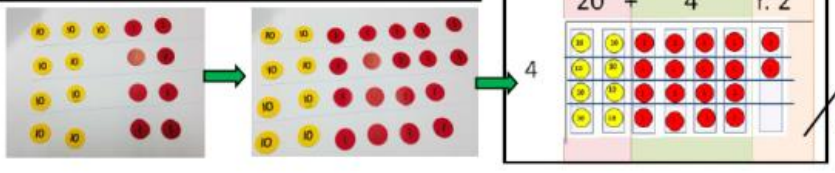
Can we divide this 100 token into 6 equal groups? , then we must exchange it for ten 10 tokens. Can we divide into 6 groups now?

Short division with exchange.



Practical experience with manipulatives is vital for children to talk through the language of division e.g. *exchange, remainder;* and to embed conceptual understanding.

Understanding remainders.



2 out of a whole group of 4 = $\frac{2}{4} = \frac{1}{2} = 0.5$

$98 \div 4 = \frac{98}{4} = 24\ r\ 2 = 24\frac{1}{2} = 24.5$

What is the same? What's different about the ways that these remainders are expressed?

Fractions

- **Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number .**
- ***Pupils connect equivalent fractions > 1 that simplify to integers with division and other fractions G 1 to division with remainders.***
- ***Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division.***
- ***Pupils should make connections between percentages, fractions and decimals***

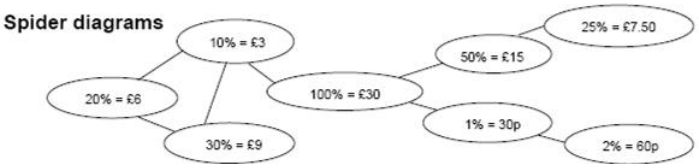
Mastery	Pupils have a firm understanding of what multiplication and division mean and have a range of strategies for dealing with large numbers, including both mental and standard written methods. They see the idea of factors, multiples and prime numbers as connected and not separate ideas to learn. They recognise how to use their skills of multiplying and dividing in new problem solving situations. Fractions and division are connected ideas: 36 ÷ 18 = 36/18 = 2; 18/36 = ½ Factors and multiples are connected ideas: 48 is a multiple of 6 and 6 is a factor of 48.

Year 6: Division

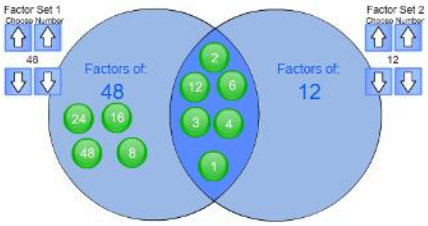
Mental calculations

Pupils should be taught to:

- **perform mental calculations, including with mixed operations and large numbers.**
- **use their knowledge of the order of operations to carry out calculations involving the four operations.**
- **identify common factors, common multiples and prime numbers.**



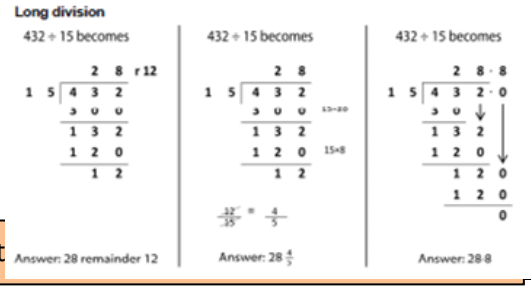
I know that 366 will divide by 6 because it has 2 and 3 as factors.



- **Solve problems involving addition, subtraction, multiplication and division**
- **use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.**

Written calculations

- **divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context**
- **divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.**
- ***Pupils practise division for larger numbers, using the formal written methods of short and long division.***



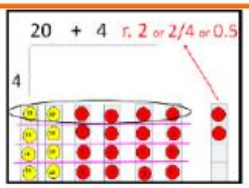
Revert to expanded methods if children find formal calculat

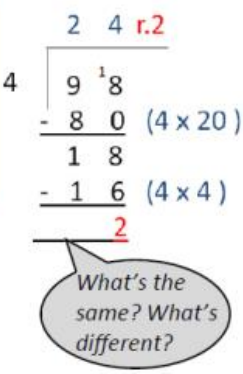
Representations to support

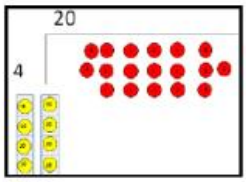
£1362.72 ÷ 40 = ?

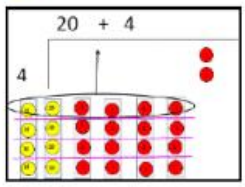
£1362.72 ÷ 4 = £340.68
[½ and ½ again.]
£340.68 ÷ 10 = £34.068
which rounds to £34.07

To introduce the long division model, use a calculation which can be represented both with manipulatives and by a short division algorithm. Use questioning and discussion to compare written methods.









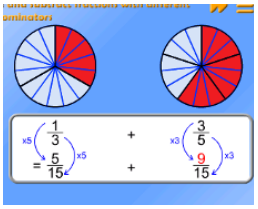
Fractions

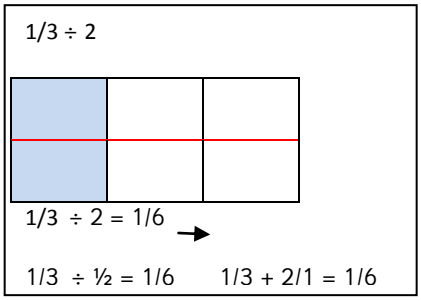
use common factors to simplify fractions, compare and order fractions, including fractions > 1

add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

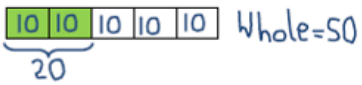
divide proper fractions by whole numbers [for example, 1/3 ÷2 = 1/6]

associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375]





2/5 of a number is 20. What is the number?



Mastery	Standard written algorithms use the conceptual structures of the mathematics to produce efficient methods of calculation. Standard written multiplication method involved a number of partial products, e.g. 36 x 24 is made up of four partial products 30 x 20, 30 x 4, 6 x 20, 6 x 4. There are connections between factors, multiples and prime numbers and between fractions, division and ratios.